Limits Worksheet Algebraically And Graphically Precalcus Questions and Answers PDF

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Part 1: Building a Foundation

What does the notation $\lim_{x \to a} f(x) = L$ signify?

Hint: Think about what happens to the function as x approaches a.

- ◯ A) $f(x)$ is undefined at $x = a$
- ◯ **B) As x approaches a, $f(x)$ approaches L ✓**
- ◯ C) $f(x)$ is always equal to L
- ◯ D) $f(x)$ is discontinuous at $x = a$

> The notation signifies that as x approaches a, the function f(x) approaches L.

Which of the following are methods to calculate limits algebraically?

Hint: Consider different algebraic techniques.

- ☐ **A) Direct substitution ✓**
- ☐ B) Graphical analysis
- ☐ **C) Factoring ✓**
- ☐ **D) Rationalization ✓**

> Methods to calculate limits include direct substitution, factoring, and rationalization.

Explain what it means for a function to be continuous at a point a.

Hint: Consider the definition of continuity.

A function is continuous at a point a if the limit as x approaches a equals the function value at a.

List two types of discontinuities that can affect the existence of a limit.

Hint: Think about removable and non-removable discontinuities.

1. Type 1

Removable discontinuity

2. Type 2

Jump discontinuity

Common types of discontinuities include removable discontinuities and jump discontinuities.

If \(\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)\), what can be concluded about \(\lim_{x \to a} f(x)\)?

Hint: Consider the implications of one-sided limits.

- A) The limit exists and equals \(f(a)\)
- **B) The limit does not exist ✓**
- C) The limit is infinite
- D) The function is continuous at \(x = a\)

If the left-hand limit does not equal the right-hand limit, then the limit does not exist.

Part 2: Understanding and Interpretation

Which statement best describes a horizontal asymptote?

Hint: Think about the behavior of functions as x approaches infinity.

- ◯ A) A line that the graph of a function approaches as \(x\) approaches a finite value
- ◯ **B) A line that the graph of a function approaches as \(x\) approaches infinity ✓**
- ◯ C) A point where the function is undefined
- ◯ D) A line that intersects the graph at multiple points

> A horizontal asymptote is a line that the graph of a function approaches as x approaches infinity.

Which of the following statements are true about limits at infinity?

Hint: Consider the behavior of functions as they grow large.

- ☐ **A) They describe the end behavior of a function ✓**
- ☐ B) They can determine vertical asymptotes
- ☐ C) They are always finite
- ☐ **D) They can be used to find horizontal asymptotes ✓**

> Limits at infinity describe the end behavior of a function and can help find horizontal asymptotes.

Describe how you would use a graph to determine if a function is continuous at a point.

Hint: Think about the visual representation of continuity.

> **To determine continuity from a graph, check if the graph is unbroken and if the limit equals the function value at that point.**

Part 3: Application and Analysis

Given $f(x) = \frac{x^2 - 1}{x - 1}$, what is $\lim_{x \to 1} f(x)$?

Hint: Consider simplifying the function before evaluating the limit.

- ◯ A) 0
- ◯ **B) 1 ✓**
- ◯ C) 2
- ◯ D) Does not exist

> The limit can be found by simplifying the function and then substituting x = 1.

Which of the following steps are necessary to find $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$?

Hint: Think about the algebraic techniques you can apply.

- ☐ A) Direct substitution
- ☐ **B) Factoring the numerator ✓**
- ☐ **C) Simplifying the expression ✓**
- ☐ D) Rationalizing the denominator

> Necessary steps include factoring the numerator and simplifying the expression.

Explain how you would apply L'Hôpital's Rule to find the limit of $\frac{\sin x}{x}$ as x approaches 0.

Hint: Consider the conditions under which L'Hôpital's Rule is applicable.

> **L'Hôpital's Rule can be applied when the limit results in an indeterminate form, such as $\frac{0}{0}$.**

Which of the following functions has a removable discontinuity at $x = 2$?

Hint: Consider the definition of removable discontinuity.

- ◯ **A) $f(x) = \frac{x^2 - 4}{x - 2}$ ✓**

- ◯ B) $f(x) = \frac{x^2 + 4}{x - 2}$
- ◯ C) $f(x) = \frac{x^2 - 4}{x^2 - 4}$
- ◯ D) $f(x) = \frac{x + 2}{x - 2}$

> A removable discontinuity occurs when a function can be redefined to make it continuous.

When analyzing the function $f(x) = \frac{1}{x}$, which of the following are true?

Hint: Consider the behavior of the function around its discontinuities.

- ☐ **A) The function has a vertical asymptote at $x = 0$ ✓**
- ☐ **B) The function is continuous for all $x \neq 0$ ✓**
- ☐ **C) The limit as x approaches 0 from the right is ∞ ✓**
- ☐ **D) The limit as x approaches 0 from the left is $-\infty$ ✓**

> The function has a vertical asymptote at x = 0 and is continuous for all x except at that point.

Part 4: Evaluation and Creation

If a function $f(x)$ has limits $\lim_{x \to a^-} f(x) = 3$ and $\lim_{x \to a^+} f(x) = 5$, what can be concluded about $\lim_{x \to a} f(x)$?

Hint: Consider the implications of one-sided limits.

- ◯ A) The limit is 4
- ◯ **B) The limit does not exist ✓**
- ◯ C) The limit is 3
- ◯ D) The limit is 5

> If the left-hand limit does not equal the right-hand limit, then the limit does not exist.

Which of the following are potential strategies to resolve an indeterminate form of type $\frac{0}{0}$?

Hint: Think about techniques used in calculus.

- ☐ A) Direct substitution
- ☐ **B) L'Hôpital's Rule ✓**
- ☐ **C) Factoring ✓**
- ☐ D) Adding a constant

> Strategies include using L'Hôpital's Rule, factoring, and simplifying the expression.

Create a real-world scenario where understanding limits is crucial, and explain how limits help solve the problem.

Hint: Think about applications of limits in real life.

> **Understanding limits can help in various fields such as physics, engineering, and economics.**