Limits Worksheet Algebraically And Graphically Precalculus Questions and Answers PDF

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Part 1: Building a Foundation

What is the notation used to represent the limit of a function $f(x)$ as x approaches a value a?

Hint: Think about the standard mathematical notation for limits.

- A) $f(a)$
- **B) $\lim_{x \to a} f(x)$ ✓**
- C) $f'(a)$
- D) $\int f(x) dx$

> The correct notation for the limit of a function as x approaches a value is $\lim_{x \to a} f(x)$.

Which of the following are methods used to evaluate limits algebraically? (Select all that apply)

Hint: Consider common algebraic techniques for finding limits.

- **A) Substitution ✓**
- **B) Factoring ✓**
- C) GraphING
- **D) Rationalizing ✓**

> Methods such as substitution, factoring, and rationalizing are commonly used to evaluate limits.

Explain what a limit represents in the context of a function's behavior.

Hint: Think about how limits describe the behavior of functions near specific points.

A limit represents the value that a function approaches as the input approaches a certain point.

List two types of discontinuities that can be identified using limits.

Hint: Consider the different ways a function can fail to be continuous.

1. Type 1

Removable Discontinuity

2. Type 2

Jump Discontinuity

Common types of discontinuities include removable discontinuities and jump discontinuities.

What is the limit of a constant function \(f(x) = c\) as \(x\) approaches any value \(a\)?

Hint: Consider the behavior of constant functions.

- A) 0
- **B) \(c\) ✓**
- C) \(a\)
- D) Undefined

The limit of a constant function as x approaches any value is the constant itself.

Part 2: Comprehension and Application

If $\lim_{x \to 3} f(x) = 7$, what does this imply about the function $f(x)$ as x approaches 3?

Hint: Think about the definition of limits and function behavior.

- ◯ A) $f(3) = 7$
- ◯ B) $f(x)$ becomes undefined at $x = 3$
- ◯ **C) $f(x)$ approaches 7 as x approaches 3 ✓**
- ◯ D) $f(x)$ has a discontinuity at $x = 3$

> This implies that as x approaches 3, the function f(x) approaches the value 7.

Which statements are true about one-sided limits? (Select all that apply)

Hint: Consider the definitions and properties of one-sided limits.

- ☐ **A) They are used to determine the behavior of a function from one direction. ✓**
- ☐ B) They are always equal to the two-sided limit.
- ☐ **C) They can help identify jump discontinuities. ✓**
- ☐ D) They are only applicable to polynomial functions.

> One-sided limits help analyze function behavior from a specific direction and can identify discontinuities.

Describe how the Squeeze Theorem can be used to find the limit of a function.

Hint: Think about how the Squeeze Theorem applies to bounding functions.

> **The Squeeze Theorem states that if a function is squeezed between two other functions that have the same limit, then it also has that limit.**

Given $f(x) = \frac{x^2 - 1}{x - 1}$, find $\lim_{x \to 1} f(x)$ using algebraic techniques.

Hint: Consider simplifying the function before evaluating the limit.

- ◯ A) 0
- ◯ B) 1
- ◯ **C) 2 ✓**
- ◯ D) Does not exist

> After simplifying, the limit can be evaluated to find the value as x approaches 1.

Part 3: Analysis, Evaluation, and Creation

Analyze the function $g(x) = \frac{x^2 - 4}{x - 2}$. What type of discontinuity does it have at $x = 2$?

Hint: Consider the behavior of the function around the point x = 2.

- ◯ A) Jump Discontinuity
- ◯ B) Infinite Discontinuity
- ◯ **C) Removable Discontinuity ✓**
- ◯ D) No Discontinuity

> The function has a removable discontinuity at x = 2 because it can be simplified.

Which of the following are true when analyzing the limit $\lim_{x \to 0} \frac{\sin x}{x}$? (Select all that apply)

Hint: Consider the properties of the sine function and its limit.

- ☐ **A) The limit is evaluated using the Squeeze Theorem. ✓**
- ☐ **B) The limit is 1. ✓**
- ☐ **C) The function is continuous at $x = 0$. ✓**
- ☐ D) The limit does not exist.

> The limit evaluates to 1 and can be shown using the Squeeze Theorem.

Analyze the behavior of the function $h(x) = \frac{1}{x}$ as x approaches 0 from the left and right. What conclusions can you draw about its limits?

Hint: Think about the behavior of the function as it approaches the point from both sides.

As x approaches 0 from the left, h(x) approaches negative infinity, and from the right, it approaches positive infinity.

Evaluate the following statement: "If \(\lim_{x \to a} f(x) = L\), then \(f(x)\) must be continuous at \(x = a\)."

Hint: Consider the definition of continuity in relation to limits.

- ◯ A) True
- ◯ **B) False ✓**
- ◯ C) It depends on the function.
- ◯ D) This statement is always true.

This statement is false; limits can exist even if the function is not continuous at that point.

Consider the real-world scenario where a car's speed is modeled by the function \(s(t) = \frac{100 t}{ t + 10}\). Evaluate the car's speed as time \(t \) approaches infinity. (Select all that apply)

Hint: Think about the behavior of rational functions as the variable approaches infinity.

- ☐ A) The speed approaches 10 units.
- ☐ **B) The speed approaches 100 units. ✓**
- ☐ **C) The speed becomes constant. ✓**
- ☐ D) The speed increases indefinitely.

As t approaches infinity, the speed approaches 100 units, indicating a constant speed.

Create a function that has a removable discontinuity at \(x = 3\) and explain how you would modify it to make it continuous.

Hint: Think about how to define a function with a hole at a specific point.

> **An example function could be \(f(x) = \frac{(x-3)(x+1)}{x-3}\), which has a removable discontinuity at x = 3. To make it continuous, redefine f(3) = 4.**

Propose two different functions that have the same limit as \(x\) approaches 2, but differ in their continuity at that point.

Hint: Consider functions that behave similarly near x = 2 but have different definitions.

1. Function 1

> f(x) = \frac{x^2 - 4}{x - 2}

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2. Function 2

> g(x) = x^2

> For example, \(f(x) = \frac{x^2 - 4}{x - 2}\) has a removable discontinuity at x = 2, while \(g(x) = x^2\) is continuous at x = 2. Both have the same limit of 4 as x approaches 2.