

Multivariable Calculus Quiz Answer Key PDF

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Provide an example of a real-world application where triple integrals are used and explain its importance.

Triple integrals are used in calculating the mass of a solid with variable density, crucial in engineering and physics for designing structures and understanding material properties.

Which of the following represents the gradient of a function $f(x, y)$?

- A. $(\partial f/\partial x, \partial f/\partial y)$ ✓**
- B. $(\partial^2 f/\partial x^2, \partial^2 f/\partial y^2)$
- C. $(f(x), f(y))$
- D. $(\partial f/\partial y, \partial f/\partial x)$

Which of the following integrals can be used to calculate volume?

- A. Double integrals ✓**
- B. Triple integrals ✓**
- C. Line integrals
- D. Surface integrals

How does the Divergence Theorem relate the flux of a vector field through a closed surface to the behavior inside the surface?

It states that the flux through a closed surface equals the integral of the divergence over the volume inside, linking surface and volume integrals.

Describe the process of converting a double integral from Cartesian to polar coordinates.

Replace x and y with $r\cos\theta$ and $r\sin\theta$, respectively. Adjust the limits of integration and include the Jacobian (r) in the integrand.

What is the result of integrating a constant function over a region?

- A. Zero
- B. The constant multiplied by the area of the region ✓**
- C. The constant
- D. Undefined

Which of the following is a critical point of the function $f(x, y) = x^2 + y^2$?

- A. (1, 1)
- B. (0, 0) ✓**
- C. (2, 2)
- D. (-1, -1)

Which of the following are applications of multiple integrals?

- A. Calculating area ✓**
- B. Determining volume ✓**
- C. Finding the center of mass ✓**
- D. Solving differential equations

What is the divergence of a vector field $F = (P, Q, R)$?

- A. $\partial P/\partial x + \partial Q/\partial y + \partial R/\partial z$ ✓**
- B. $\partial Q/\partial x + \partial R/\partial y + \partial P/\partial z$
- C. $\partial P/\partial y + \partial Q/\partial z + \partial R/\partial x$
- D. $\partial P/\partial z + \partial Q/\partial x + \partial R/\partial y$

Which of the following are valid coordinate systems for multivariable calculus?

- A. Cartesian ✓**
- B. Polar ✓**
- C. Cylindrical ✓**

D. Spherical ✓

What is the significance of the curl of a vector field in physical applications?

The curl measures the rotation or swirling strength of a vector field, important in fluid dynamics and electromagnetism.

Explain how to find the critical points of a function of two variables.

1. Compute the first partial derivatives of the function with respect to each variable. 2. Set each partial derivative equal to zero to form a system of equations. 3. Solve the system of equations to find the critical points.

Discuss the role of Lagrange multipliers in optimization problems with constraints.

Lagrange multipliers help in solving optimization problems with constraints by introducing additional variables (the multipliers) that account for the constraints, allowing us to find the extrema of a function while satisfying those constraints.

What is the primary use of Lagrange multipliers?

- A. To find the divergence of a vector field
- B. To solve differential equations
- C. To find local maxima and minima of functions subject to constraints ✓**
- D. To compute line integrals

In which coordinate system is the point (r, θ, z) used?

- A. Cartesian
- B. Polar
- C. Cylindrical ✓**
- D. Spherical

Which theorem relates a line integral around a closed curve to a double integral over the region it encloses?

- A. Stokes' Theorem
- B. Green's Theorem ✓**

- C. Divergence Theorem
- D. Fundamental Theorem of Calculus

Which theOREMS are used to convert between different types of integrals?

- A. Green's Theorem ✓
- B. Stokes' Theorem ✓
- C. Divergence Theorem ✓
- D. Fundamental Theorem of Calculus

In the context of vector fields, which statements are correct?

- A. A vector field assigns a vector to every point in space. ✓
- B. The curl of a vector field measures its tendency to rotate. ✓
- C. The divergence of a vector field measures its tendency to spread out. ✓
- D. Vector fields can only exist in two dimensions.

What is the partial derivative of $f(x, y) = x^2y$ with respect to x ?

- A. $2xy$ ✓
- B. x^2
- C. y
- D. $2x$

Which of the following are true about the gradient vector?

- A. It points in the direction of maximum increase of the function. ✓
- B. It is perpendicular to level curves. ✓
- C. It is a scalar quantity.
- D. It can be used to find critical points. ✓