Linear Algebra Quiz Questions and Answers PDF

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Which operation is not defined for vectors?

- ◯ Addition
- ◯ Scalar multiplication
- ◯ **Division ✓**
- ◯ Dot product

> The operation that is not defined for vectors is division. While vectors can be added, subtracted, and multiplied (by scalars or other vectors in specific ways), there is no standard way to divide one vector by another.

Which of the following matrices is an identity matrix?

- ◯ **\(\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}\) ✓**
- ◯ \(\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}\)
- ◯ \(\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}\)
- ◯ \(\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix}\)

> An identity matrix is a square matrix with ones on the diagonal and zeros elsewhere. For example, the 2x2 identity matrix is represented as [[1, 0], [0, 1]].

What is the rank of a zero matrix?

- ◯ **0 ✓**
- ◯ 1
- ◯ Depends on the size of the matrix
- ◯ Infinity

> The rank of a zero matrix is always zero, regardless of its dimensions. This is because a zero matrix does not have any linearly independent rows or columns.

Explain how linear algebra is utilized in machine learning algorithms.

> Linear algebra is utilized in machine learning algorithms through the use of vectors and matrices to represent data, perform operations like matrix multiplication for transformations, and apply techniques such as Singular Value Decomposition (SVD) for dimensionality reduction.

What are the conditions under which a matrix is invertible?

> A matrix is invertible if it is square and its determinant is non-zero.

Explain the significance of the rank-nullity theorem in linear algebra.

> The rank-nullity theorem states that for any linear transformation from a finite-dimensional vector space V to a vector space W, the dimension of V (dim V) is equal to the rank of the transformation (dim of the image) plus the nullity of the transformation (dim of the kernel), expressed as: dim V = rank + nullity.

Which of the following is a necessary condition for a matrix to be invertible?

- ◯ It must be a square matrix. ✓

- ◯ It must be a diagonal matrix.
- ◯ It must be a symmetric matrix.
- ◯ It must be a zero matrix.

> A necessary condition for a matrix to be invertible is that it must be square, meaning it has the same number of rows and columns. Additionally, the determinant of the matrix must be non-zero.

Describe the process of the Gram-Schmidt orthogonalization and its purpose.

> **The Gram-Schmidt orthogonalization process involves taking a set of linearly independent vectors and systematically constructing an orthogonal set from them. This is done by iteratively subtractively projecting each vector onto the previously constructed orthogonal vectors, ensuring that the resulting vectors are orthogonal to each other.**

Which of the following are true about eigenvectors?

- ☐ **They are always non-zero. ✓**
- ☐ **They can be scaled by any non-zero scalar. ✓**
- ☐ They are orthogonal to each other.
- ☐ **They correspond to eigenvalues. ✓**

> Eigenvectors are non-zero vectors that change only by a scalar factor when a linear transformation is applied to them. They are associated with eigenvalues, which represent the factor by which the eigenvector is scaled.

Which matrices are diagonalizable?

- ☐ **Identity matrix ✓**
- ☐ **Zero matrix ✓**
- ☐ Any square matrix
- ☐ **Symmetric matrix ✓**

> A matrix is diagonalizable if it has enough linearly independent eigenvectors to form a basis for the vector space. Specifically, an n x n matrix is diagonalizable if it has n distinct eigenvalues or if its algebraic multiplicity equals its geometric multiplicity for each eigenvalue.

Which of the following are applications of linear algebra?

- [ ] **Computer graphics ✓**
- [ ] **Quantum mechanics ✓**
- [ ] **Financial modeling ✓**
- [ ] **Language processing ✓**

> Linear algebra is widely used in various fields such as computer graphics, machine learning, engineering, and data analysis, among others.

How do eigenvalues and eigenvectors contribute to understanding the stability of a system?

> **Eigenvalues with negative real parts indicate stability, while positive real parts indicate instability.**

Discuss the role of linear transformations in computer graphics.

> **Linear transformations play a crucial role in computer graphics by providing a mathematical framework for transforming geometric objects in a consistent and efficient manner, facilitating operations like scaling, rotation, and translation.**

What is the eigenvalue of the identity matrix of size 3x3?

- ◯ 0
- ◯ **1 ✓**
- ◯ 2
- ◯ 3

> The eigenvalues of the identity matrix are all equal to 1. For a 3x3 identity matrix, this means it has three eigenvalues, each equal to 1.

What is the determinant of the matrix \(\begin{matrix} 3 & 4 \\ 2 & 1 \end{matrix}\)?

- ◯ **-5 ✓**
- ◯ 5
- ◯ 10
- ◯ -10

> The determinant of a 2x2 matrix can be calculated using the formula \(ad - bc\), where \(a, b, c, d\) are the elements of the matrix. For the matrix \(\begin{matrix} 3 & 4 \\ 2 & 1 \end{matrix}\), the determinant is calculated as \(3*1 - 4*2 = 3 - 8 = -5\).

What is the dimension of a vector space defined by the set of all 2x2 matrices?

- ◯ 2
- ◯ 3
- ◯ **4 ✓**
- ◯ 5

> The dimension of the vector space defined by the set of all 2x2 matrices is 4, as each matrix can be represented by 4 independent entries.

Which of the following are properties of a vector space?

- ☐ **Closure under addition ✓**
- ☐ **Closure under scalar multiplication ✓**
- ☐ **Existence of a zero vector ✓**
- ☐ Existence of a multiplicative inverse

> A vector space is defined by a set of vectors that can be added together and multiplied by scalars, satisfying specific properties such as closure, associativity, and the existence of an additive identity and inverses.

Which of the following statements are true about orthogonal matrices?

- [ ] **Their transpose is equal to their inverse. ✓**
- [ ] **They preserve vector norms. ✓**
- [ ] **They are always square matrices. ✓**
- [ ] Their determinant is always zero.

> Orthogonal matrices are square matrices whose rows and columns are orthogonal unit vectors, meaning their dot product is zero and their norms are one. They have the property that their inverse is equal to their transpose, and they preserve vector lengths and angles during transformations.

Which of the following vectors is orthogonal to \(\begin{matrix} 1 \\ 2 \end{matrix}\)?

- ◯ \(\begin{matrix} 2 \\ 1 \end{matrix}\)
- ◯ **\(\begin{matrix} -2 \\ 1 \end{matrix}\) ✓**
- ◯ \(\begin{matrix} 1 \\ -2 \end{matrix}\)
- ◯ \(\begin{matrix} 0 \\ 0 \end{matrix}\)

> A vector is orthogonal to another if their dot product is zero. For the vector \(\begin{matrix} 1 \\ 2 \end{matrix}\), any vector of the form \(\begin{matrix} -2 \\ 1 \end{matrix}\) or \(\begin{matrix} 2 \\ -1 \end{matrix}\) will be orthogonal to it.

Which of the following are methods to solve a system of linear equations?

- [ ] **Gaussian elimination ✓**
- [ ] **Matrix inversion ✓**
- [ ] Cross product
- [ ] **Substitution method ✓**

> There are several methods to solve a system of linear equations, including substitution, elimination, and matrix methods such as Gaussian elimination or using the inverse of a matrix.