Eigenvalues and Eigenvectors Quiz Answer Key PDF

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Which method is commonly used to find eigenvalues of large matrices?

A. Simplex method
B. Power iteration ✓
C. Gaussian elimination
D. Newton's method

A matrix A is diagonalizable if it can be expressed as:

A. A = PDP^{-1} ✓
B. A = P + D + P^{-1}
C. A = DPD^{-1}
D. A = P^{-1}DP

What is the geometric multiplicity of an eigenvalue?

A. The number of times an eigenvalue appears in the characteristic polynomial
B. The number of linearly independent eigenvectors for an eigenvalue ✓
C. The determinant of the matrix
D. The trace of the matrix

In which field is Principal Component Analysis (PCA) commonly used?

A. Sorting algorithms
B. Principal Component Analysis ✓
C. Network routing
D. Encryption

Explain how eigenvalues are used to determine the stability of a dynamical system.

In a dynamical system, the eigenvalues of the system's matrix determine stability; if all eigenvalues have negative real parts, the system is stable, indicating that perturbations will decay over time.

Explain what an eigenvector is and how it relates to an eigenvalue.

An eigenvector is a non-zero vector that, when a linear transformation is applied, changes only in scale by a scalar factor known as the eigenvalue.

Describe the process of deriving the characteristic equation for a 2x2 matrix.

To derive the characteristic equation for a 2x2 matrix A, subtract λ times the identity matrix from A, then set the determinant of the resulting matrix to zero.

What are true statements about the algebraic multiplicity of an eigenvalue? (Select all that apply)

A. It is always equal to the geometric multiplicity.
B. It can be greater than the geometric multiplicity. ✓
C. It is the number of times an eigenvalue appears as a root. ✓
D. It is always less than the geometric multiplicity.

In stability analysis, a system is considered stable if all eigenvalues have:

A. Positive real parts
B. Negative real parts ✓
C. Zero real parts
D. Imaginary parts only

Complex eigenvalues indicate which of the following in a system? (Select all that apply)

A. Oscillatory behavior ✓
B. Stability
C. Rotational dynamics ✓
D. Linear growth

Which numerical methods are used to compute eigenvalues and eigenvectors? (Select all that apply)

A. QR algorithm ✓

B. Power iteration ✓
C. Gradient descent
D. Simplex method

Why are eigenvectors often normalized?

A. To simplify calculations
B. To ensure they have a unit length ✓
C. To change their direction
D. To make them orthogonal

Discuss the difference between algebraic and geometric multiplicity of an eigenvalue.

Algebraic multiplicity is the number of times an eigenvalue appears as a root of the characteristic polynomial, while geometric multiplicity is the number of linearly independent eigenvectors associated with the eigenvalue.

What conditions must be met for a matrix to be diagonalizable?

A matrix is diagonalizable if it has enough linearly independent eigenvectors to form a basis for the space, meaning the sum of the geometric multiplicities equals the dimension of the matrix.

What is an eigenvalue?

A. A vector that does not change direction under a linear transformation
B. A scalar that scales an eigenvector under a linear transformation ✓
C. A matrix that transforms a vector
D. A determinant of a matrix

Eigenvalues and eigenvectors are used in which of the following fields? (Select all that apply)

A. Quantum mechanics ✓
B. Image processing ✓
C. Weather forecasting
D. Financial modeling ✓

How are eigenvalues and eigenvectors used in Principal Component Analysis (PCA)?

In Principal Component Analysis (PCA), eigenvalues and eigenvectors are used to determine the principal components of the data, where eigenvectors indicate the directions of maximum variance and eigenvalues quantify the amount of variance captured by each principal component.

Which equation is used to find eigenvalues?

A. A\mathbf{v} = \lambda\mathbf{v}

B. \det(A - \lambda I) = 0 ✓

C. A = PDP^{-1}

D. \mathbf{v} = \lambda A\mathbf{v}

Which components are used to derive the characteristic equation? (Select all that apply)

A. Matrix A ✓

B. Identity matrix I ✓

C. Eigenvalue λ ✓

D. Eigenvector v

Which of the following statements are true about eigenvectors? (Select all that apply)

A. They can be zero vectors.

B. They change direction under a linear transformation.

C. They can be scaled to have a unit length. ✓

D. They correspond to eigenvalues. ✓